Ad-hoc Networking – Models and Methods

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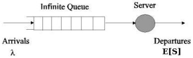
Part III

Queueing Models

Queues

- Important class of .
- Queueing phenomena occuring in .
- Examples
  - queues in front of counters,
  - computer systems with a (shared resources),
  - call handling in a station.
- Modelling issues
  - time between ,
  - population,
  - amount of waiting room,
  - amount of a customer requires,
  - number of available,
  - strategy.
- Terms and used interchangeably.

Graphical Representation – Notation



- Parameter
  - λ of per time unit,
  - $E[S]$.
- Values of interest
  - $E[N]$ system,
  - $E[N_q]$ queue,
  - $E[W]$ until customer gets served,
  - $E[R]$, i. e., overall time a customer spends in system.

Kendall Notation

- Describes queueing stations in unambigous way.
- | | | | |
  - : customer arrival process,
  - : customer service process requirements,
  - : number of servers,
  - : maximum number of customers,
  - : size of the customer population,
  - : implemented scheduling strategy.

Modelling Issues

- : described by random variables
  - M: Markovian,
  - G: General,
  - ...
- : finite or inifinite number of server entities.
- : finite or infinite number of allowed customers.
- : finite or infinite number of available customers.
-
  - First Come First Serve,
  - Shortest Job Next,
  - Round Robin,
  - Priority Scheduling.

Setup

- Relates to
  - ,
  - .
- Arriving job either
  - , no other job in the system, or
  - has to wait until all jobs .

Understanding the Law

- Queueing system as , at in time, arrival rate .
- $E[R]$
  - jobs arrived more than $E[R]$ time units ago: ( ) be gone;
  - jobs arrived within the past $E[R]$ time units: ( ) present in the system.
- : $\lambda \cdot E[R]$ jobs.
- $E[N] = \lambda \cdot E[R]$.

⇨ Think of .

- No losses!
  - System equals λ.

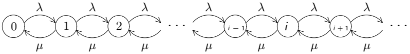
Breaking the Black Box

- Little's Law on finer levels of abstraction.
-
  - , λ jobs arrive,
  - $E[W]$ waiting time, ,
  - $E[N_q] = \lambda \cdot E[W]$ jobs in queue.
-
  - , λ jobs arrive,
  - $E[S]$ time in the server,
  - $E[N_s] = \lambda \cdot E[S]$ jobs in server.
- Remark: $E[N_s]$ average time server is busy. .

General Remarks on Little's Law

- No assumptions about
  .
- Only values.
- Little's Law independent of
  - number of ,
  - used .

Analysing the M|M|1 Queue

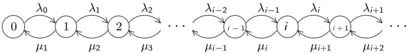


- M|M|1 means
  - interarrival ,
  - service time rate ,
  - one server,
  - queue of ,
  - infinite population, i. e., infinitely many jobs.
- Determine .
- Check
  - $\rho = \frac{\lambda}{\mu}$.

Results

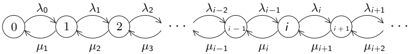
- $E[N_q] = \lambda \cdot E[W]$, $E[N] = \lambda \cdot E[R]$, $E[R] = E[W] + E[S]$.
- (oisson rrivals ee ime verages) property: $E[W] = E[N]E[S]$.
- .
- $E[W] =$.
- $E[N] =$.
- $E[N_q] =$.

Transition Diagram – Generator Matrix



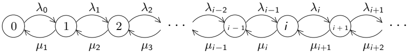
- λ_i .
- μ_i .
- Steady-state probabilities via Q, or
- via .

Global Balance Equations



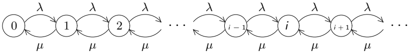
- Assumption: system will reach .
- Probability into a state equals probability .
- State 0: .
- State i: .
- .

Solving the GBEs



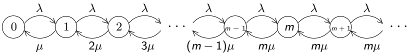
- General: to say if explicit solution exists.
- .
- .
- $p_i = p_0 \prod_{j=0}^{i-1} \frac{\lambda_j}{\mu_{j+1}}, i = 1, 2, \ldots$.
- p_0 can be calculated. (Try at home.)

Measures of Interest



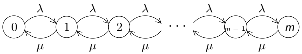
- .
- utilisation.
- .
- .
- $E[W]$, $E[N_q]$: .

Multiserver Systems



- Arrival rate: .
- Service rate:
  - $\mu_i = i\mu, i = 0, 1, \ldots, m,$
  - $\mu_i = m\mu, i = m+1, m+2, \ldots.$
- $\rho = \frac{\lambda}{m\mu}$.
  - $m\rho = \frac{\lambda}{\mu}$.
- Steady-state from GBEs
  - $p_i = p_0 \frac{(m\rho)^i}{i!}, i = 0, 1, \ldots, m-1,$
  - $p_i = p_0 \frac{(m\rho)^m}{m!} \frac{(m\rho)^{i-m}}{m^{i-m}}, i = m, m+1, \ldots.$

Bounded Buffer of Size m



- Arrival rate
  - $\lambda_i =$,
  - $\lambda_i =$.
- Service rate
  - $\mu_i =$,
  - $\mu_i =$.
- Throughput X
  - ,
  - .
- Utilisation .

Generally Distributed Service Times

M|G|1 queues.

- .
  - No memoryless distribution.
  - of a job.
- of service time distribution.

G|G|1 queues.

- Exact result of .
- More practical: .

Overview

- Combination of single .
- Number of customers not known .
-
  - Series of queues.
  - General FFQNs, .
- .
-
  - Arrival process not necessarily .
  - Approximate results.

Overview

- of customers.
- queueing networks
- queues.
- given.
- Underlying CTMC can be solved directly.
- More convenient methods
  - .
  - .
  - .

M|M|1 Queue – Constant Rates

- Calculate $E[W]$ and $E[N_q]$.
- What is the probability to have at least k jobs in the system?

CTMC

- Draw the finite CTMC for the depicted GNQN.
- Use triples (i, j, k) to indicate the number of customers in each node.